HW2 Q1, Q2

Tuesday, February 14, 2012
9:02 AM

① $f_c = 900\text{ MHz} = 9\times10^8\text{ Hz}$, $R = 100\text{ m}$

$\Downarrow$

worst $d = 100\text{ m}$

Need to make sure that the terminals at the cell boundary receive the minimum required power.

nondirectional antenna $\Rightarrow G_{T_x}G_{R_x} = 1$

By the Friss Equation,

$$\frac{10\mu W}{P_t} = \frac{P_r}{P_t} = \left(\frac{\sqrt{G_{T_x}G_{R_x}}\, c}{4\pi d f}\right)^2 = \left(\frac{3\times\cancel{10^8}}{4\pi\times100\times\cancel{9}_{3}\times\cancel{10^8}}\right)^2$$

$$P_t = 10\times10^{-6}\times(12\pi\times100)^2 = 144\pi^2\times10^{-1} \approx 142\text{ W}$$

If the system frequency is changed to $f = 5\text{ GHz}$
$= 5\times10^9\text{ Hz}$,

then we need

$$P_t = \frac{P_r}{\left(\frac{\sqrt{G_{T_x}G_{R_x}}\, c}{4\pi d f}\right)^2} = \frac{10\times\cancel{10^{-6}}}{\left(\frac{3\times\cancel{10^8}}{4\pi\times\cancel{100}\times5\times\cancel{10^9}_{10}}\right)^2}$$

$$= \frac{10}{9}\times(4\pi\times5)^2 \approx 4.39\text{ kW}$$

② Simplified path loss model:

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^\gamma$$

In dB, this is

$$P_r[dB] - P_t[dB] = \underbrace{10\log_{10}K}_{b} + \gamma\,\underbrace{10\log_{10}\left(\frac{d_0}{d}\right)}_{\alpha} \quad \star$$

For free-space path gain, $K = \left(\frac{\lambda}{4\pi d_0}\right)^2 = \left(\frac{c}{4\pi d_0 f}\right)^2$

Here, $f = 900\text{ MHz}$, $d_0 = 1\text{ m}$

Therefore,

$$\underbrace{10\log_{10} K}_{b} = 10\log_{10}\left(\frac{c}{4\pi d_0 f}\right)^2 \approx -31.53 \text{ dB}$$

Note that ☆ is of the form

$$y(x) = b + \gamma x$$

We are given five pairs of y_i, x_i.

Want to find γ such that

$$MSE = \sum_{i=1}^{5}\left(y(x_i) - y_i\right)^2 = \underbrace{\sum_i \left(b + \gamma x_i - y_i\right)^2}_{\text{parabola on } \gamma}$$

is minimized.

So, we find

$$\frac{d}{d\gamma} MSE = \sum_i 2\left(b + \gamma x_i - y_i\right) x_i$$

$$\downarrow$$

$$0 = b\sum_i x_i + \gamma \sum_i x_i^2 - \sum_i x_i y_i$$

$$\text{So, } \gamma = \frac{\sum_i x_i y_i - b\sum_i x_i}{\sum_i x_i^2} \approx 3.71$$

y	d	x
-70	10	-10
-75	20	-13
-90	50	-17
-110	100	-20
-125	300	-24.77

At $d = 150$ m, $x = 10\log_{10}\left(\frac{d_0}{d}\right) \approx -21.76$.

$$\Downarrow$$

$$y = b + \gamma\alpha \approx -112.24$$

So, $P_r[dB] - P_t[dB] = -112.24$

$$P_r[dB] = P_t[dB] - 112.24$$

$$P_r[dBm] = \underbrace{P_t[dBm]}_{0} - 112.24 = -112.24 \text{ dBm.}$$

Remark:

If you haven't played with dB and dBm often, you probably find it strange that my answer above does not have the conversion of the unit of -112.24 to dBm.

This is because it is not power. It is simply a number that represents the factor of gain/attenuation.

To see this, let's try an easy example.
Consider two values of power:

$$P_1 = 100 \text{ W} \quad \text{and} \quad P_2 = 100{,}000 \text{ W}$$

Then,

$$P_1 = 10 \log_{10} 100 \text{ dB} = 20 \text{ dB}$$

$$= 10 \log_{10} \frac{100}{1m} \text{ dBm} = 50 \text{ dBm}$$

Similarly,

$$P_2 = 10 \log_{10} 10^5 \text{ dB} = 50 \text{ dB}$$

$$= 10 \log_{10} \frac{10^5}{1m} \text{ dBm} = 80 \text{ dBm}$$

Nothing strange so far...

Now, note that

$$P_2 = 1000 \times P_1$$

In dB, we have

$$50\,dB \rightarrow P_2[dB] = 10\log_{10} 1000 + P_1[dB] \leftarrow 20\,dB.$$

$$= 30[dB] + P_1[dB]$$

The number 1,000 is unitless. It is not a quantity that represents power.

Now, note that in dBm, we have

$$\underbrace{P_2[dBm]}_{80\,dBm} = \underbrace{30[dB]}_{\text{still!!}} + \underbrace{P_1[dBm]}_{50\,dBm}$$

To avoid confusion, you may see some references use

[dBW] (or [dB(W)]) and [dBmW] (or [dB(mW)])

for the quantities that really represent power.

In which case, we write

$$P_1[dBW] = 30[dB] + P_2[dBW]$$

and

$$P_1[dBmW] = 30[dB] + P_2[dBmW].$$

still have no "W" because they do not represent power.

Summary: It's OK to directly add or subtract dB values to a power level in dBm. The final answer will be a power level in dBm.

HW2 Q3

Wednesday, January 25, 2012
4:31 PM

Let f_i be the center freq. of the i^{th} band group.

The Friss Equation says

$$\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}}\, c}{4\pi}\right)\left(\frac{1}{df}\right)^2 = \frac{K}{d^2 f^2}$$

At f_1, the range (max distance) is $d_1 = 10$ m.

So, the min amount of P_r required for the system to work is $P_r = \frac{K}{(d_1 f_1)^2} P_t$.

Now, at f_i, assuming that P_t is the same, then at distance d, the received power is

$$P_r = \frac{K}{(d f_i)^2} P_t.$$

So, to have P_r of at least $\frac{K}{(d_1 f_1)^2} P_t$, which is the min received power for the system to work, we need

$$\frac{\cancel{K}}{(d f_2)^{\cancel{2}}} \cancel{P_t} \geq \frac{\cancel{K}}{(d_1 f_1)^{\cancel{2}}} \cancel{P_t}$$

$$d \leq \underbrace{\frac{d_1 f_1}{f_i}}$$

so, this is the range d_i. (max distance)

Hence,

$$d_i = \frac{\overset{10}{d_1}\,\overset{3960}{f_1}}{f_i}$$

f_i	d_i
5.544	7.14

f_i	d_i
5,544	7.14
7,128	5.56
8,712	4.55
10,032	3.95

Note that in [Nan, Guo, Qiu, Mo, and Takahashi, 2007], the d_i's are incorrectly calculated by $d_i = d_1 \left(\frac{f_1}{f_i}\right)^2$ which gives 5.10, 3.09, 2.07, and 1.56 respectively.

HW2 Q4-Q10

Wednesday, December 02, 2009

2:39 PM

(4) Recall that $c = f\lambda$, which means $\lambda = \frac{c}{f}$

Here, $f = 0.9\times10^9$, 1.9×10^9, and 5.8×10^9 Hz.

Hence, $\lambda = 33.3$, 15.8, and 5.17 cm

(5) We will use MATLAB to find the values of N when i and j are between 0 and 7.

Here is the code:

```
[I J] = meshgrid(0:6,0:6);
N = I.^2 + I.*J + J.^2;
N = unique(reshape(N, 1, numel(N)));
N = N(N > 7);
N = N(1:15)
```

This part finds the unique values of N

Take only N > 7

Use only 15 values.

So, the next 15 values of N are

9	12	13	16	19	21	25	27	28	31	36	37	39	43	48

We know that we can't have any missing values of N between the above numbers because we have consider all i, j between 0 and 6. Any other values of N must come from (i, j) pair which has at least one of the i or j ≥ 7 which will give $N \geq 7^2 = 49$.

(6) (a) Each simplex channel use 25 kHz.

So, each duplex channel use $25\times2 = 50$ kHz.

Total spectrum = 20 MHz

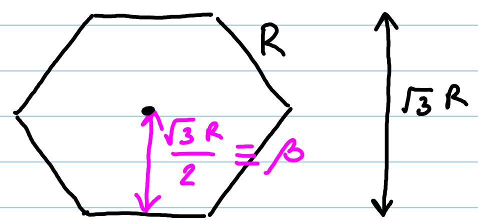
duplex channel $= \frac{2\cancel{0}\times10^{\cancel{6}\,3}}{5\cancel{0}\times\cancel{10^3}} = \boxed{400 \text{ channels}}$

(b) Each cluster will use to whole 400 channels.

These channels are divided among the cells in each cluster.
For $N=4$, there are 4 cells in a cluster.
Hence

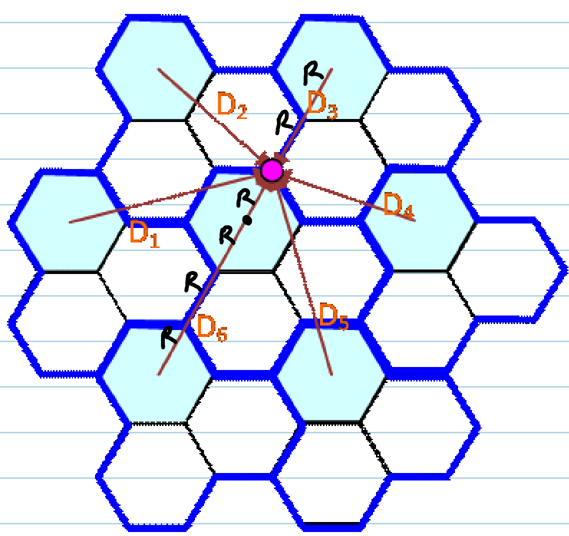
$$\#\text{ channel} = \frac{400}{4} = \boxed{100 \text{ channels per cell site.}}$$

(7) (a) To find the distance $D_1, \ldots, D_6$, let's recall some facts about hexagon.

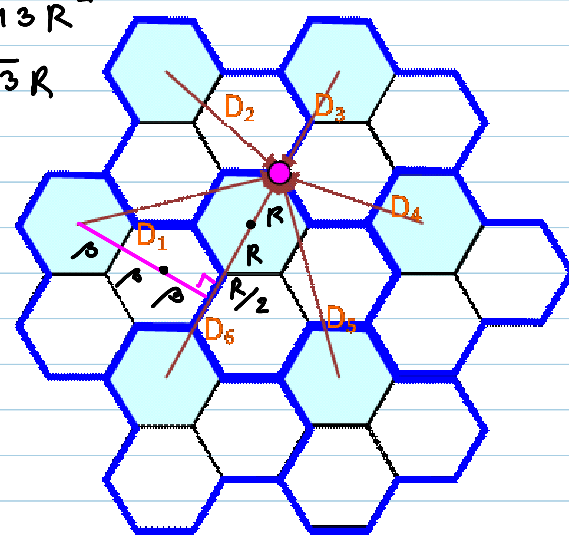


D_3 and D_6 are easy to find. $D_3 = R + R = 2R$
$D_6 = R + R + R + R = 4R$

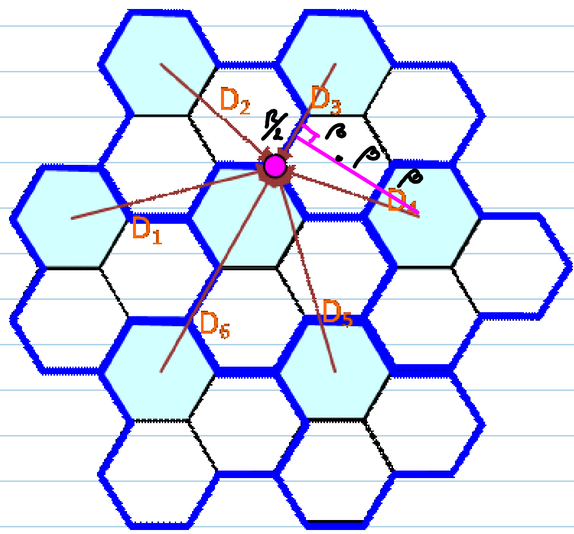
For the rest of the distances, the key to find them is to select suitable right triangles.



$$D_1^2 = D_5^2 = (3\beta)^2 + \left(\frac{5}{2}R\right)^2 = \left(9 \times \frac{3}{4} + \frac{25}{4}\right)R^2$$
$$= 13R^2$$
$$D_1 = D_5 = \sqrt{13}\,R$$



$$D_2^2 = D_4^2 = \left(\frac{R}{2}\right)^2 + (3\beta)^2 = \left(\frac{1}{4} + 9 \times \frac{3}{4}\right)R^2$$
$$= 7R^2$$
So, $D_2 = D_4 = \sqrt{7}\,R$



So,

$D_1 = D_5 = \sqrt{13}\,R$
$D_2 = D_4 = \sqrt{7}\,R$
$D_3 = 2R$
$D_6 = 4R$

(b) $$\frac{S}{I} = \frac{R^{-\gamma}}{\sum_{i=1}^{6} D_i^{-\gamma}} = \frac{1}{\sum_{i=1}^{6}\left(\frac{D_i}{R}\right)^{-\gamma}} = \frac{1}{2\times(\sqrt{13})^{-4} + 2\times(\sqrt{7})^{-4} + 2^{-4} + 4^{-4}}$$

$$= \boxed{8.399} = 10\log 8.399 \text{ dB}$$
$$= \boxed{9.242 \text{ dB}}$$

(c) $$\frac{D}{R} = \sqrt{3N} \Rightarrow D = \sqrt{3N}\times R = \sqrt{3\times 3}\,R \quad (N=3)$$
$$= \boxed{3R}$$

$$\frac{S}{I} = \frac{1}{6\times 3^{-4}} = \boxed{13.5} = 10\log 13.5 \text{ dB}$$
$$= \boxed{11.303 \text{ dB}}$$

(d) In part (c) we use approximated distances and hence the answer is different from part (b) which use the exact distances.

When N is large, the difference will be small.

8 $$\frac{S}{I} = \frac{1}{K}\left(\sqrt{3N}\right)^4 = \frac{1}{K}(3N)^2 = \frac{1}{K}9N^2$$

We need this number to be $\geq 15 \text{ dB} = 10^{\frac{15}{10}} = 10^{\frac{3}{2}}$

Recall that we want N to be small to get large capacity value. Hence, we need to pick minimal value of N such that the above inequality is still satisfied.

For (a), we use $K = 6$.

$$\frac{S}{I} = \frac{1}{\cancel{6}_2} \times \cancel{9}^3 \times N^2 \geq 15\text{ dB}$$

$$N^2 \geq \frac{2}{3} \times 10^{\frac{3}{2}}$$

Possible values of N are 3 4 7 9 ....

$$N \geq \sqrt{\frac{2}{3} \times 10^{3/2}} = 4.591$$

From Q_1, the min value of N such that it is still ≥ 4.6 is $\boxed{N = 7.}$

For (b), we use $K = 2$.

$$\frac{S}{I} = \frac{1}{2} \times 9N^2 \geq 10^{3/2}$$

$$N \geq \sqrt{\frac{2}{9} \times 10^{3/2}} = 2.651$$

From Q_1, the min value of N such that it is still ≥ 2.6 is $\boxed{N = 3.}$

For (c), we use $K = 1$

$$\frac{S}{I} = 9N^2 \geq 10^{3/2}$$

$$N \geq \sqrt{\frac{1}{9} \times 10^{3/2}} = 1.874$$

From Q_1, the min value of N such that it is still ≥ 1.874 is $\boxed{N = 3}$

So, by using 120° sectoring the capacity of the system increases from the case of omnidirectional antenna.

However, if we've already use 120° sectoring, using 60° sectoring does not help in term of capacity!!

using 60° sectoring does not help in term of capacity!!

(9) Let $\text{ErlangB}(m, A) = \dfrac{A^m/m!}{\sum_{i=0}^{m} A^i/i!}$

This gives the probability of blocking (P_b).

Of course, we want P_b to be small.
In this question, we want $P_b \le \frac{0.5}{100} = 0.005$.

For fixed m, ErlangB(m, A) is an increasing function of A. Hence, if we don't want P_b to be greater than some value, we will need to limit the value of A to be less than some max quantity as well.

(a) $m = 5 \Rightarrow P_b = \text{ErlangB}(5, A) \le 0.005$

$\downarrow$ MATLAB

$A \lesssim 1.13$ Erlangs

Each user generates 0.1 Erlangs.

So n users will generate $n \times 0.1$ Erlangs.

Hence, we need $n \times 0.1 \le 1.13$

$n \lesssim 11.3$

So, the system can support $\boxed{\text{11 users}}$

(b) $m = 15 \Rightarrow \text{Erlang B}(15, A) \le 0.005$

$A \lesssim 7.38$

$\Rightarrow n \lesssim 73.8$

So, the system can support $\boxed{\text{73 users}}$

(c) $m = 25 \Rightarrow \text{ErlangB}(25, A) \le 0.005$

$A \lesssim 15$

$\Rightarrow n \lesssim 150$

So, the system can support $\boxed{\text{150 users}}$

$\Rightarrow n \leq 150$

So, the system can support $\boxed{150 \text{ users}}$

(10) $\lambda = 3$ calls per hour

$\frac{1}{\mu} = 5$ minutes $= \frac{5}{60}$ hour $= \frac{1}{12}$ hour.

(a) $A_u = \frac{\lambda}{\mu} = 3 \times \frac{1}{12} = \boxed{\frac{1}{4} \text{ Erlang}}$ per user

(b) Erlang B $(1, A) \leq 0.01$

$$\Rightarrow \underset{n \times A_u}{\underset{\parallel}{A}} \leq 0.01$$

$$n \leq \frac{0.01}{1/4} = 0.04$$

So, the system can support $\boxed{0 \text{ user}}$

Note that this calculation comes from our assumption of M/M/m/m queue which assumes "infinite" number of users with extremely small Erlang per user.

Of course, we never have "infinite" number of users in real system.

So, it will not give an accurate answer for this case.

Intuitively, the system which has one channel should be able to support at least 1 user with 0% blocking.

The Erlang B formula should become more accurate when there are a lot of users.

(c) Erlang B $(5, A) \leq 0.01$

$$\Rightarrow \underset{n \times A_u}{\underset{\parallel}{A}} \leq 1.36$$

$n \leq 4 \times 1.36 = 5.44$

So, the system can support $\boxed{5 \text{ users}}$

(d) $\text{Erlang B}\left(5, 2 \times 5 \times \frac{1}{4}\right) = 0.0697 = \boxed{6.97\%}$